Anglais philosophique – « Paradoxes » – Textes

1 Sorites Paradoxes / Paradoxes of vagueness

1.0.1

Suppose that a movie camera is focused on a tadpole confined in a small bowl of water. The camera runs continuously for three weeks, and at the end of that time there is a frog in the bowl. At 24 frames per second we will have, assuming that the camera works perfectly, 43,545,600 pictures. Let these be arranged in a series S in the order in which they were taken. Then consider this property of numbers : the property of being the number of a picture in series S which is such that the creature shown in the picture is, at the time the picture was made, a tadpole. It seems clear that however doubtful the ascription of this property may be in some cases, it is correctly ascribable to 1 and not correctly ascribable to 43,545,600. That is, the first picture depicts a tadpole and the last does not. These two apparently unquestionable facts may be expressed symbolically, letting "P" stand for the property of numbers just described, as follows :

A $P(1)$

B $\neg P(43,545,600)$.

However, from A and B, it is easy to derive, using the classical version of the least number principle,

C $(\exists n)(P(n) \wedge \neg(Pn+1))$.

(The classical version of the least number principle is that if the number 1 has a certain predicate and a larger number n does not, then there is a least number among the set of numbers between 1 and n which do not have the predicate. See e.g. Kleene 1952, p. 190.)

Why should this be paradoxical ? Well, on the meaning we have given to "P", C means that there is some picture in the series S such that it is a picture of a tadpole, while the very next picture, taken one twenty-fourth of a second later, is not a picture of a tadpole. So it would seem to follow that the creature depicted was a tadpole, and then a split second later, was not. And this argument is obviously independent of the speed of the camera, which could be taken right up to the theoretical limit of camera speed, so that the split second in which the thing depicted ceases to be a tadpole would be a very short time indeed. And this strikes many people as extremely counterintuitive. Many people, even many with at least some grasp of logic, would accept A and B but reject C, which follows from A and B by simple rules of logic.

J. Cargile, « The Sorites Paradox », *British Journal for the Philosophy of Science*, 20 (1969).

1.0.2

At the core of classical (i.e. standard) logic and semantics is the principle of bivalence, according to which every statement is either true or false. This is the principle most obviously threatened by vagueness. When, for example, did Rembrandt become old ? For each second of his life, one can consider the statement that he was old then. Some of those statements are false ; others are true. If all of them are true or false, then there was a last second at which it was false to say that Rembrandt was old, immediately followed by a first second at which it was true to say that he was old. Which second was that ? We have no way of knowing. Indeed, it is widely felt to be just silly to suppose that there was such a second. Our use of the word 'old' is conceived as too vague to single one out. On such grounds, the principle of bivalence has been rejected for vague languages. To reject bivalence is to reject classical logic or semantics.

At some times, it was unclear whether Rembrandt was old. He was neither clearly old nor clearly not old. The unclarity resulted from vagueness in the statement that Rembrandt was old. We can even use such examples to define the notion of vagueness. An expression or concept is vague if and only if it can result in unclarity of the kind just exemplified. Such a definition does not pretend to display the underlying nature of the phenomenon. In particular, it does not specify whether the unclarity results from the failure of the statement to be true or false, or simply from our inability to find out which. The definition is neutral on such points of theory.

T. Williamson, *Vagueness*, Routledge, 1994.

1.0.3

Imagine a patch darkening continuously from white to black. At each moment during the process the patch is darker than it was at any earlier moment. Darkness comes in degrees. The patch is dark to a greater degree than it was a second before, even if the difference is too small to be discriminable by the naked eye. Given that there are as many moments in the interval of time as there are real numbers between 0 and 1, there are at least as many degrees of darkness as there are real numbers between 0 and 1, an uncountable infinity of them. Such numbers can be used to measure degrees of darkness. Now at the beginning of the process, the sentence 'The patch is dark' is perfectly false, for the patch is white. At the end, the sentence is perfectly true, for the patch is black. In the middle, the sentence is true to just the degree to which the patch is dark. Truth comes in degrees. For 'The patch is dark' to be true just is for the patch to be dark ; for 'The patch is dark' to be true to a certain degree just is for the patch to be dark to that degree. Even if we cannot discriminate between all these degrees in practice, we have made the truth of our sentence depend on a property which does in fact come in such degrees. Thus there are at least as many degrees of truth as there are degrees of darkness, and so at least as many as there are real numbers between 0 and 1, an uncountable infinity of them. Such numbers can be used to measure degrees of truth. So the thought goes.

T. Williamson, *Vagueness*, Routledge, 1994.

Perhaps the vagueness of a language consists in its capacity in principle to be made precise in more than one way. Not every substitution of precise meanings for vague ones counts as making the language precise, of course. Rather, vague meanings are conceived as incomplete specifications of reference. To make the language precise is to complete these specifications without contradicting anything in their original content. For example, the meaning of 'heap' and the non-linguistic facts are supposed to determine of some things that they are heaps, of others that they are not heaps, and of still others to leave the matter open. The clear cases are those of the first kind, the clear non-cases those of the second, and the borderline cases those of the third. To make 'heap' precise is to assign it a meaning that makes it true of the clear cases, false of the clear non-cases, and either true or false of the borderline cases.

T. Williamson, *Vagueness*, Routledge, 1994.

No one knows whether I am thin. I am not clearly thin ; I am not clearly not thin. The word 'thin' is too vague to enable an utterance of 'TW is thin' to be recognized as true or as false, however accurately my waist is measured and the result compared with vital statistics for the rest of the population. I am a borderline case for 'thin'. If you bet someone that the next person to enter the room will be thin, and I walk through the door, you will not know whether you are entitled to the winnings. Suppose that an utterance of 'TW is thin' is either true or false. Then since we do not know that TW is thin and do not know that TW is not thin, we are ignorant of something. Either 'TW is thin' expresses an unknown truth, or 'TW is not thin' does. We do not even have an idea how to find out whether TW is thin, given my actual measurements and those of the rest of the population. Arguably, we cannot know in the circumstances that TW is thin or that TW is not thin ; in that sense, we are necessarily ignorant of something. Most work on vagueness has taken it for granted that these consequences are absurd. It therefore rejects the original supposition that an utterance of 'TW is thin' is either true or false. Borderline cases are held not to involve ignorance, on the grounds that there is no fact of the matter for us to know, hence nothing for us to be ignorant of. On this view, vague utterances in borderline cases are not bivalent.

T. Williamson, *Vagueness*, Routledge, 1994.

How is the principle of bivalence to be understood ? It does not say that everything is either true or false, for no one supposes that a drop of water is true or false. The principle does not even apply to every meaningful sentence, or use of one to perform a speech act, for there is no need to suppose that a question or command is true or false. Nor does it apply to every well-formed declarative sentence. If a teacher pronounces 'He was there then' as a sample sentence of English, leaving 'he', 'there' and 'then' undetermined in reference, nothing has been said to be the case, truly or falsely. The principle of bivalence claims truth or falsity when, and only when, something has been said to be the case. To say that something is the case, in the relevant sense, is not always to assert that it is. The notions of truth and falsity apply to suppositions as well as assertions. If something true is said by an utterance of 'Not P', it is because something false is said, but not asserted, by the component utterance of 'P'. The same applies to other truth-functors. Bivalence is often formulated with respect to the object of the saying, a proposition (statement, . . .). The principle then reads : every proposition is either true or false. However, on this reading it does not bear very directly on problems of vagueness. A philosopher might endorse bivalence for propositions, while treating vagueness as the failure of an utterance to express a unique proposition. On this view, a vague utterance

> in a borderline case expresses some true propositions and some false ones (a form of supervaluationism might result). There is no commitment to a bivalent classification of utterances, or to the ignorance on our part that such a classification implies. The problem of vagueness is a problem about the classification of utterances. To debate a form of bivalence in which the truth-bearers are propositions is to miss the point of the controversy. In a relevant form of bivalence, the truth-bearers are (perhaps with a little artificiality) the utterances themselves. The principle is explicitly restricted to occasions when someone uses an utterance to say that something is the case, in brief (if again with a little artificiality), when the utterance says that something is the case. The principle may be formulated as a schema :
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> (B) If *u* says that P, then either *u* is true or *u* is false.
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> In (B), '*u*' is to be replaced by a name of an utterance and 'P' by a declarative sentence whose inscription says that something is the case. Since the utterance named is presumably not a constituent of the relevant instance of (B), it need not be in English. Since the sentence in place of 'P' is a constituent of that instance, it must be in English. Although the notion of saying in (B) is not perfectly precise, it is precise enough for present purposes.

T. Williamson, *Vagueness*, Routledge, 1994.

1.1 Références

1. R. Sorensen, « Vagueness », *The Stanford Encyclopedia of Philosophy*,
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3. R. Keefe & P. Smith (éd.), *Vagueness : A Reader*, MIT Press, 1996.
4. R. M. Sainsbury, *Paradoxes*, 3rd Edition, Cambridge University Press, 2009, ch. 3.